Singularities of Poisson structures and Hamiltonian bifurcations

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Consider a Poisson structure on $C^\infty(\mathbf{R}^3, \mathbf{R})$ with bracket $\{,\}$ and suppose that C is a Casimir function. Then $\{f, g\} =< \nabla C, (\nabla g \times \nabla f) >$ is a possible Poisson structure. This confirms earlier observations concerning the Poisson structure for Hamiltonian systems that are reduced to a one degree of freedom system and generalizes the Lie-Poisson structure on the dual of a Lie algebra and the KKS-symplectic form. The fact that the governing reduced Poisson structure is described by one function makes it possible to find a representation, called the energy-momentum representation of the Poisson structure, describing both the singularity of the Poisson structure and the singularity of the energy-momentum mapping and hence the bifurcation of relative equilibria for such systems. It is shown that Hamiltonian Hopf bifurcations are directly related to singularities of Poisson structures of type $\mathfrak{sl}(2)$.