Differentiability of the volume between levels of a function, from an application of optimization in Radiotherapy

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In the presentation I will introduce some of the relevant essentials of Radiotherapy and concerning the optimization process explain the role of differentiability of the volume function. To be more precise the latter is about the dependence on one of the levels of the volume of a region enclosed by the levelsets of a function. It is perhaps not surprising that critial values of this last function play a crucial role. I will treat this subject in some more detail and give a few examples.

Radiotherapy is based on the fact that healthy cells are more resistive to radiation damage than tumor cells. Despite that careful choices are necessary to obtain a dose distribution in the patient that meets criteria for eradicating a malignant tumour and at the same time avoiding critical damage to healthy tissues and organs. This has led to the construction of object functions that formalize these objectives. By optimizing such functions with respect to treatment parameters, one hopes to get optimal treatment plans.

Relating treatment parameters to criteria on the dose distribution in the patient is a non-trivial task. To simplify matters one does not consider the 3-dimensional dose distribution, but a derived quantity namely the dose-volume histogram for each relevant region (tumor, healthy organ). This is essentially a function that gives the volume of a region that receives at least a specified dose. The object functions depend on treatment parameters via this volume function. Since one uses a Newton method to find an optimum, differentiability of the volume function becomes important.