Singularities on the boundary of the stability domain near 1:1 resonance

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We study the linear differential equation x' = Lx in 1:1 resonance. That is, L is a 4 by 4 matrix with eigenvalues (ib,-ib,ib,-ib). We wish to find the stability domain in gl(4,R), the space of 4 by 4 matrices. Moreover we wish to find the singularities of the boundary of the stability domain.

The 1:1 resonance turns up in many applications, ranging from fluid dynamics and wave phenomena to rotating mechanical devices. Such systems are frequently considered as perturbed Hamiltonian, reversible or equivariant systems. In many examples the latter turn up at the boundary of the stability domain, especially at the singularities. Therefore determining the stability of perturbations can be delicate.

Since a neighborhood of L in gl(4,R) is 16-dimensional we put some effort in reducing the dimension. Here keywords are equivalence classes and transversality. In several steps we are able to reduce to a 3-sphere that contains all information about the neighborhood of L. The boundary of the stability domain is contained in two right conoids. The singularities of this surface are transverse self-intersections, Whitney umbrellas and intersections of self-intersections. A Whitney stratification allows us to describe the neighborhood of L and identify the stability domain.