Towards Theory of Limit Cycles

Valery Gaiko

Belarusian State University of Informatics and Radioelectronics
Minsk, Belarus

We establish the global qualitative analysis of planar polynomial dynamical systems and suggest a new geometric approach to solving Hilbert's Sixteenth Problem on the maximum number and relative position of their limit cycles in two special cases of such systems. First, using geometric properties of four field rotation parameters of a new canonical system, we present a proof of our earlier conjecture that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is $(3:1)$. Then, by means of the same geometric approach, we solve the Problem for Liénard's polynomial system (in this special case, it is considered as Smale's Thirteenth Problem). Besides, generalizing the obtained results, we present a solution of Hilbert's Sixteenth Problem on the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and, applying the Wintner–Perko termination principle for multiple limit cycles, we develop an alternative approach to solving the Problem. By means of this approach, for example, we give another proof of the main theorem for a quadratic system and complete the global qualitative analysis of a generalized Liénard's cubic system with three finite singularities which is very important for applications. Finally, we establish the global qualitative analysis of a cubic centrally symmetric dynamical system which can be used as a learning model of planar neural networks and a Liénard-type piecewise linear dynamical system which is well-known in radio-electronics.